Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Student number\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Assignment 4**

Derive the expressions of linear strain components , , and  of the polar coordinate system. Use the displacement representation  where the components depend on the polar coordinates  and . Use definitions

, , , .

**Solution template**

In manipulation of vector expression containing vectors and tensors, it is important to remember that tensor (), cross (), inner () products are non-commutative (order may matter). The basis vectors of a curvilinear coordinate system are not constants which should be taken into account if gradient operator is a part of expression. Otherwise, simplifying an expression or finding a specific form in a given coordinate system is a straightforward (sometimes tedious) exercise. For simplicity of presentation, outer (tensor) products like  are denoted by . Otherwise, the usual rules of algebra apply: Gradient operator  acts on everything on its right hand side, the operator is treated like a vector etc.

Let us start with the gradient of displacement (an outer product). Substitute first the representations in the polar coordinate system

.

Then expand to have a term-by-term representation. Keep the order of the basis vectors and the position of derivatives



Use the derivative rule of products. Notice that the basis vectors are not constants and may have non-zero derivatives

 .

Substitute the derivatives of the basis vectors

 .

Combine the terms having the same pair of basis vectors (order matters so terms containing  and  cannot be combined)

.

Conjugate of a second order tensor can be obtained by swapping the basis vectors in all the pairs. Conjugate is a kind of transpose and can also be obtained by transposing the matrix of the component representation.



Finally using the definition 

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In the components of strain , ,  and , indices are in the same order as the indices in the basis vector pairs. Hence

, , . 🡸